





UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD

CLASS - 9

Question Paper Code : UM9264

KEY

1	2	3	4	5	6	7	8	9	10
В	В	С	С	В	С	С	В	С	D
11	12	13	14	15	16	17	18	19	20
А	С	В	В	А	D	А	С	С	С
21	22	23	24	25	26	27	28	29	30
D	D	D	А	В	А	А	В	С	А
31	32	33	34	35	36	37	38	39	40
A,D	A,B,C,D	A,B	A,B,D	A,C,D	D	С	А	С	В
41	42	43	44	45	46	47	48	49	50
A	В	С	В	D	С	A	A	В	В

EXPLANATIONS

MATHEMATICS - 1

01. (B)
$$\sqrt{(3+\sqrt{2})(12-\sqrt{32})}$$

= $\sqrt{(3+\sqrt{2})(12-4\sqrt{2})}$
= $\sqrt{(3+\sqrt{2})(4)(3-\sqrt{2})}$
= $\sqrt{4\times [3^2-(\sqrt{2})^2]}$
= $2\sqrt{7}$

02. (B) LHS = $\left[\frac{1}{(x-5)(x-3)} + \frac{1}{(x-3)(x-1)} - \frac{2}{(x-5)(x-1)}\right]$ $\left[\frac{(x-1) + (x-5) - 2(x-3)}{(x-1)(x-3)(x-5)}\right]$ $= \frac{2x - 6 - 2x + 6}{(x-1)(x-3)(x-5)}$ = 0

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03. (C) Given
$$3\pi r^2 = 115.5 \text{ cm}^2$$

 $3 \times \frac{22}{7} \times r^2 = \frac{231}{2} \text{ cm}^2$
 $r^2 = \frac{231}{2} \text{ cm}^2 \times \frac{7}{22} \times \frac{1}{3}$
 $r = \sqrt{\frac{49}{4}} \text{ cm}$
 $r = \frac{7}{2} \text{ cm}$
Volume $= \frac{2}{3} \pi r^3$
 $= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^3$
 $= \frac{539}{6} \text{ cm}^3$
04. (C) Given
 $s - a = 60 \text{ cm}, s - b = 15 \text{ cm} \text{ & s - c = 5 \text{ cm}}$
 $\therefore s - a + s - b + s - c = (60 + 15 + 5) \text{ cm}$
 $3s - 2s = 80 \text{ cm}$
 $s = 80 \text{ cm}$
Area of ΔABC
 $= \sqrt{s(s-a)(s-b)(s-c)}$
 $\sqrt{80 \times 60 \times 15 \times 5}$
 $\sqrt{20 \times 4 \times 20 \times 3 \times 5 \times 3 \times 5}$
 $= 20 \times 2 \times 3 \times 5 \text{ cm}^2$
 $= 600 \text{ cm}^2$
05. (B) $(2\sqrt{2} + 3\sqrt{3})^2$
 $(2\sqrt{2})^2 + 2 \times 2\sqrt{2} + 3\sqrt{3} + (3\sqrt{3})^2$
 $= 8 + 12\sqrt{6} + 27$
 $= 35 + 12\sqrt{6}$

06. (C)
$$\frac{1}{\sqrt{7} + \sqrt{6} - \sqrt{13}}$$
$$= \frac{1}{(\sqrt{7} + \sqrt{6}) - (\sqrt{13})} \times \frac{(\sqrt{7} + \sqrt{6}) + \sqrt{13}}{(\sqrt{7} + \sqrt{6}) + (\sqrt{13})}$$
$$= \frac{\sqrt{7} + \sqrt{6} + \sqrt{13}}{(\sqrt{7} + \sqrt{6})^2 - (\sqrt{13})^2}$$
$$= \frac{\sqrt{7} + \sqrt{6} + \sqrt{13}}{7 + 6 + 2\sqrt{42} - 13}$$
$$= \frac{\sqrt{7} + \sqrt{6} + \sqrt{13}}{13 + 2\sqrt{42} - 13}$$
$$= \frac{\sqrt{7} + \sqrt{6} + \sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$$
$$= \frac{\sqrt{7} + \sqrt{6} + \sqrt{13}}{2\sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$$
$$= \frac{\sqrt{7} \times \sqrt{42} + \sqrt{6} \times \sqrt{42} + \sqrt{13} \times \sqrt{42}}{2(\sqrt{42})^2}$$
$$= \frac{7\sqrt{6} + 6\sqrt{7} + \sqrt{546}}{2\times 42}$$
$$= \frac{7\sqrt{6} + 6\sqrt{7} + \sqrt{546}}{84}$$
07. (C) Given $3x + 7^\circ + 2x - 19^\circ + x = 180^\circ$ $6x = 180^\circ + 12^\circ$ $x = \frac{192^\circ}{6} = 32^\circ$
$$\therefore \quad \angle \text{COD} = 2x - 19^\circ = 64^\circ - 19^\circ = 45^\circ$$
08. (B) $x^2 - y^2 + z^2 - p^2 - 2yp - 2zx$
$$= (x^2 + z^2 - 2zx) - (y^2 + p^2 + 2yp)$$
$$= (x - z)^2 - (y + p)^2$$
$$= (x - z + y + p)(x - z - y - p)$$

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cm

09. (C) LHS =
$$6x(x^2 - 4y^2) - 3y(x^2 - 4y^2)$$

= $(x^2 - 4y^2)(6x - 3y)$
= $(2 - 2y)(x + 2y)(3)(2x - y)$
= $3(2x - y)(x + 2y)(x - 2y)$
10. (D) Given $(x + 1)$ is a factor of $p(x)$
= $x^{2023} - 3x^{2022} + k$
 $p(-1) = 0$
 $p(-1) = (-1)^{2023} - 3(-1)^{2022} + k = 0$
 $-1 - 3 + k = 0$
 $k = 4$
11. (A) Given $(2x - 3)$ is a factor of $p(x)$
= $2x^3 - x^2 + mx + n$
 $\therefore p\left(\frac{3}{2}\right) = 0$
 $2\left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2 + m\left(\frac{3}{2}\right) + n = 0$
 $\Rightarrow 2 \times \frac{27}{8} - \frac{9}{4} + \frac{3m}{2} + n = 0$
 $\frac{27 - 9 + 6m + 4n}{4} = 0$
 $18 + 6m + 4n = 0 \times 4$
 $6m + 4n = -18$
 $2(3m + 2n) = -18$
 $3m + 2n = -9$
12. (C) Given $x = -3$ and $y = 4$ is the solution of $5x + 3y = k$
 $\therefore 5(-3) + 3(4) = k$
 $-15 + 12 = k$
 $k = -3$

13. (B) It is a right angled triangle of base 8 units and height 7 units





= 28 square units

14. (B) Consider the following figure



Using the exterior angle theorem, we get

$$\angle y = \angle 1 + \angle 2 \qquad \dots \dots (1)$$
$$\angle 1 + \angle x = \angle 4 \qquad \dots \dots (2)$$

Add equations (1) and (2)

 $\angle y + \angle 1 + \angle x = \angle 1 + \angle 2 + \angle 4$

$$\Rightarrow \angle x + \angle y = \angle 2 + \angle 4$$

Now, since $\angle 2 = \angle 4 = \frac{z}{2}$, x + y = z

$$\Rightarrow x = z - y$$

15. (A) Consider the figure



Since thickness of the bowl is 0.25 cm, the outer radius R of the bowl is 5 + 0.25= 5.25 cm

$$V_{inner} = \frac{2}{3}\pi r^{3} = \frac{2}{3}\pi (5)^{3} cm^{3}$$

 \therefore Volume of steel used = V_{outer} - V_{inner}

$$=\frac{2}{3}\pi(5.25)^{3}-\frac{2}{3}\pi(5)^{3}$$
$$=\frac{2}{3}\times3.14\times(5.25^{3}-5^{3})$$
$$=41.25 \text{ cm}^{3}$$

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19. (C)
$$a = \sqrt{5}, b = \sqrt[3]{7} \text{ and } c = \sqrt[4]{36}$$

 $a = 5^{\frac{1}{2}}, b = 7^{\frac{1}{3}} \text{ and } c = (6^2)^{\frac{1}{4}} = 6^{\frac{1}{2}}$
 $a = (5^3)^{\frac{1}{6}}, b = ((7)^2)^{\frac{1}{6}} \text{ and } c = (6^3)^{\frac{1}{6}}$
 $a = (125)^{\frac{1}{6}}, b = (49)^{\frac{1}{6}} \text{ and } c = (216)^{\frac{1}{6}}$
 $\therefore b < a < c$
20. (C) $\sqrt{4a^2 + 25b^2 + 49c^2 - 20ab + 70bc - 28ca}$
 $= \sqrt{(2a)^2 + (-5b)^2 + (-7c)^2 + 2(2a)(-5b) + 2(-5b)(7c) + 2(-7c)(2a)}$
 $= \sqrt{(2a - 5b - 7c)^2}$
 $= (2a - 5b - 7c)$
21. (D) Given ∠EAD = 20° ⇒ ∠EOD = 2∠EAD = 40°
D



But $\triangle EOD \cong \triangle COD$ [:: Given DE = DC] $\Rightarrow \angle COD = \triangle EOD = 40^{\circ}$

 $\therefore \qquad \angle BOD = \angle BOD + \angle COD = 100^\circ + 40^\circ = 140^\circ$

$$\therefore \angle BAD = \frac{\angle BOD}{2} = \frac{140^{\circ}}{2} = 70^{\circ}$$

22. (D) Let ABCD be the field in the form of a trapezium in which AB || CD such that



AB =25 m, BC = 13 cm, CD = 10 cm and DA = 14 m

Draw CE || DA and CF \perp EB

Clearly, ABCE is a parallelogram. CE = DA = 14 m and AE = CD = 10 m... EB = AB - AE = (25 - 10) m = 15 m.... In Δ EBC, we have EB = 15 m, BC = 13 m and CE = 14 m a = 15 m, b = 13 m and c = 14 m \therefore s = $\frac{1}{2}$ (15 + 13 + 14) m = 21 m (s - a) = (21 - 15) m = 6 m.... (s - b) = (21 - 13) m = 8 m and(s - c) = (21 - 14) m = 7 m \therefore area (Δ EBC) = $\sqrt{s(s-a)(s-b)(s-c)}$ $=\sqrt{21\times6\times8\times7}$ m² $=\sqrt{7\times3\times3\times2\times2\times4\times7}$ m² $= (7 \times 3 \times 2 \times 2) m^2 = 84 m^2$ Also area (Δ EBC) $=\left(\frac{1}{2} \times EB \times CF\right) = \left(\frac{1}{2} \times 15m \times CF\right)$ $\therefore \frac{1}{2} \times 15 \text{ m} \times \text{CF} = 84 \text{ m}^2$ \Rightarrow CF = $\frac{84 \times 2}{15}$ m = $\frac{56}{5}$ m = 11.2 m CF = 11.2 m ÷ Area (trap. ABCD) = $\frac{1}{2} \times (AB + CD) \times CF$ $=\left\{\frac{1}{2} \times (25+10) \times 11.2\right\} m^{2}$ = (35 × 5.6) m² = 196 m² Hence, area of trapezium ABCD is 196 m² 23. (D) Radius of each hemispherical end = 7 cm



Hence of each hemispherical part = its radius = 7 cm

Height of the cylindrical part

= (104 – 2 × 7) cm = 90 cm

Area of surface to be polished

- = 2(curved surface area of the hemisphere)
- + (curved surface area of the cylinder)
- = $[2(2\pi r^2) + 2\pi rh]$ sq. units

$$= \left[\left(4 \times \frac{22}{7} \times 7 \times 7 \right) + \left(2 \times \frac{22}{7} \times 7 \times 90 \right) \right] \mathrm{cm}^{2}$$
$$= (616 + 3960) \mathrm{cm}^{2} = 4576 \mathrm{cm}^{2}$$

$$=\left(\frac{4576}{10\times10}\right)$$
 dm² = 45.76 dm²

[:: 10 cm = 1 dm]

 \therefore cost of polishing the surface of the solid

= ₹ (45.76 × 10) = ₹ 457.60

24. (A) $30^{\circ} + \alpha_1 = \alpha, 45^{\circ} + \alpha_2 = \beta$



Adding $(30^\circ + 45^\circ) + (\alpha_1 + \alpha_2) = \alpha + \beta$ $\Rightarrow 75^\circ + 55^\circ$ $= x^\circ \Rightarrow x = 130^\circ$

25. (B) $\angle ACD + 80^\circ = 180^\circ$ $\Rightarrow \angle ACD = 100^\circ$ $\angle ECF = \angle ACD = 100^\circ$ $100^\circ + 25^\circ + \angle CEF = 180^\circ$ $\Rightarrow \angle CEF = 55^\circ$

26. (A) We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore AP = \frac{1}{2}AB = \left(\frac{1}{2} \times 8\right) cm = 4 cm$$

$$\therefore AP = \frac{1}{2}AB = \left(\frac{1}{2} \times 8\right) cm = 4 cm$$

$$CQ = \frac{1}{2}CD = \left(\frac{1}{2} \times 6\right) cm = 3 cm$$
Join OA and OC
Then, OA = OC = 5 cm
From the right-angled AOPA, we have
 $OP^{2} = OA^{2} - AP^{2} = [(5)^{2} - (4)^{2}] cm^{2} = 9 cm^{2}$

$$\Rightarrow OP = 3 cm$$
From the right-angled ΔOQC , we have
 $OQ^{2} = OC^{2} - CQ^{2} = [(5)^{2} - (4)^{2}] cm^{2} = 9 cm^{2}$

$$\Rightarrow OP = 3 cm$$
From the right-angled ΔOQC , we have
 $OQ^{2} = OC^{2} - CQ^{2} = [(5)^{2} - (3)^{2}] cm^{2} = 16 cm^{2}$

$$\Rightarrow OQ = 4 cm$$
Since $OP \perp AB, OQ \perp CD and AB || CD, the points P, OQ are collinear.
$$\therefore PQ = OP + OQ = (3 + 4) cm = 7 cm$$
27. (A) Since ACE is a striaght line, we have
 $\angle ACB + \angle BCE = 180^{\circ}$
 $\Rightarrow 90^{\circ} + \angle BEC = 180^{\circ}$
 $\Rightarrow 2BCC = 90^{\circ}$
Also, $\angle DBC = \frac{1}{2} \angle COD = \left(\frac{1}{2} \times 40^{\circ}\right) = 20^{\circ}$
[angle at centre = 2 × angle at a point on a circle]
 $\Rightarrow \angle EBC = \angle DBC = 20^{\circ}$
Now, in $\angle EBC$, we have
 $\angle EBC + \angle BCE = 180^{\circ}$
 $\Rightarrow 20^{\circ} + 2BCE = 180^{\circ}$
 $\Rightarrow 20^{\circ} + 90^{\circ} + \angle CED = 180^{\circ}$
 $\Rightarrow 20^{\circ} + 90^{\circ} + \angle CED = 180^{\circ}$
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 $\Rightarrow 20^{\circ} + 90^{\circ} + \angle CED = 180^{\circ}$
 $\Rightarrow 20^{\circ} + 90^{\circ} + \angle CED = 180^{\circ}$
 $\Rightarrow 2CED = 180^{\circ} - 110^{\circ} = 70^{\circ}$
Hence, $\angle CED = 70^{\circ}$
 $\therefore x + \frac{1}{x} = \left(\frac{5 - \sqrt{21}}{2}\right) + \left(\frac{5 + \sqrt{21}}{2}\right)$
 $= \frac{5 - \sqrt{21} + 5 + \sqrt{21}}{2} = 5$
 $\Rightarrow x^{2} + \frac{1}{x^{2}} + 2 \times xx + \frac{1}{x} = 25$$

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$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) + 2 = 25 \Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 25 - 2 = 23$$
And, $x + \frac{1}{x} = 5 \Rightarrow \left(x + \frac{1}{x}\right)^3 = (5)^3 = 125$

$$x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x} \left(x + \frac{1}{x}\right) = 125$$

$$\Rightarrow \left(x^3 + \frac{1}{x^3}\right) + 3 \times 5 = 125$$

$$\Rightarrow \left(x^3 + \frac{1}{x^3}\right) = 125 - 15 = 110$$

$$\therefore \left(x^3 + \frac{1}{x^3}\right) - 5\left(x^2 + \frac{1}{x^2}\right) + \left(x + \frac{1}{x}\right)$$

$$= 110 - 5 \times 23 + 5 = 110 - 115 + 5$$

$$= 115 - 115 = 0$$

MATHEMATICS - 2

31. (A, D)

 $\boldsymbol{\pi}$ is irrational

 \Rightarrow Surface area and volume of the sphere also irrational numbers

32. (A, B, C, D)

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Option (A) : If a + b + c = 0, then a^3 + b^3 + c^3 = 3abc

Option (B) : Given a + b + c = 0

squaring on both sides (a + b + c)^2 = 0

a^2 + b^2 + c^2 + 2(ab + bc + ca) = 0

\therefore a^2 + b^2 + c^2 = -2(ab + bc + ca)

Option (C) : Given a + b + c = 0

\therefore c = -(a + b)

If a + b + c = 0, then

a^3 + b^3 + c^3 = 3abc = -3ab(a + b)

Option (D) : Given a + b + c = 0

\therefore (a + b)^2 = (-c)

squaring on both sides

(a + b)^2 = (-c)^2

a^2 + b^2 + 2ab = c^2

\therefore a^2 + b^2 - c^2 = -2ab
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33. (A, B) Given $f(x) = 2x^{100} - 19x^{99} + 8x^{98} + 19x^{95} - 10x^{92}$ If f(x) is divided by (x - 1) then the remainder is f(1) : $f(1) = 2 \times 1^{100} - 19(1)^{99} + 8(1)^{98} + 19(1)^{95} - 10(1)^{92}$ = 2 - 19 + 8 + 19 - 10= 0 $f(1) = 0 \implies (x - 1)$ is a factor of f(x) $f(-1) = 2(-1)^{100} - 19(-1)^{99} + 8(-1)^{98} + 19(-1)^{95} - 10(-1)^{92}$ $2 \times 1 - 19 \times -1 + 8(1) + 19(-1) - 10(1)$ = 2 + 19 + 8 - 19 - 10 $f(1) = 0 \Longrightarrow (x + 1)$ is a factor of f(x) $f(-2) = 2(-2)^{100} - 19(-2)^{99} + 8(-2)^{98} + 19(-2)^{95} - 10(-2)^{92}$ $= 2 \times 2^{100} + 19 \times 2^{99} + 8 \times 2^{98} - 19 \times 2^{95} - 10 \times 2^{92}$ \therefore f(-2) \neq 0 \Rightarrow (x + 2) is not a factor of f(x) Similarly (x - 2) is not a factor of f(x)34. (A, B, D) Every square is also a parallelogram Every rectangle is also a parallelogram Every rhombus is also a parallelogram 35. (A, C, D) Option A × $\sqrt[5]{a^8b^7c^4} = \sqrt[5]{a^7b^3c} \times \sqrt[5]{a^8b^7c^4}$ $\sqrt[5]{a^7 \times a^8 \times b^3 \times b^7 \times c \times c^4}$ $\sqrt[5]{a^{15}h^{10}c^5}$ $= a^{3}b^{2}c$ which is a rational number $\therefore \sqrt[5]{a^7b^3c}$ is the RF of $\sqrt[5]{a^8b^7c^4}$ Option B × $\sqrt[5]{a^8b^7c^4} = \sqrt{a^2b^3c} \times \sqrt[5]{a^8b^7c^4}$ Orders of both surds are not same. If we made orders are same also we cannot get rational number. $\therefore \sqrt{a^2 b^3 c}$ is not a RF of $\sqrt[5]{a^8 b^7 c^4}$ Similarly we can prove $\sqrt[5]{a^2b^8c^6}$ and $\sqrt[5]{a^7b^8c^{11}}$ are also RF of $\sqrt[5]{a^8b^7c^4}$

