Unified International
Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD



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## MATHEMATICS - 1

1. (B) $\sqrt{(3+\sqrt{2})(12-\sqrt{32})}$
$=\sqrt{(3+\sqrt{2})(12-4 \sqrt{2})}$
$=\sqrt{(3+\sqrt{2})(4)(3-\sqrt{2})}$
$=\sqrt{4 \times\left[3^{2}-(\sqrt{2})^{2}\right]}$
$=2 \sqrt{7}$

## EXPLANATIONS

2. (B) $\mathrm{LHS}=\left[\frac{1}{(x-5)(x-3)}+\frac{1}{(x-3)(x-1)}-\frac{2}{(x-5)(x-1)}\right]$

$$
\begin{aligned}
& {\left[\frac{(x-1)+(x-5)-2(x-3)}{(x-1)(x-3)(x-5)}\right]} \\
& =\frac{2 x-6-2 x+6}{(x-1)(x-3)(x-5)} \\
& =0
\end{aligned}
$$

3. (C) Given $3 \pi \mathrm{r}^{2}=115.5 \mathrm{~cm}^{2}$
$3 \times \frac{22}{7} \times \mathrm{r}^{2}=\frac{231}{2} \mathrm{~cm}^{2}$
$\mathrm{r}^{2}=\frac{231}{2} \mathrm{~cm}^{2} \times \frac{7}{22} \times \frac{1}{3}$
$r=\sqrt{\frac{49}{4}} \mathrm{~cm}$
$r=\frac{7}{2} \mathrm{~cm}$
Volume $=\frac{2}{3} \pi r^{3}$
$=\frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \mathrm{~cm}^{3}$
$=\frac{539}{6} \mathrm{~cm}^{3}$
$=89 \frac{5}{6} \mathrm{~cm}^{3}$
4. (C) Given
$s-a=60 \mathrm{~cm}, \mathrm{~s}-\mathrm{b}=15 \mathrm{~cm} \& \mathrm{~s}-\mathrm{c}=5 \mathrm{~cm}$
$\therefore \quad s-a+s-b+s-c=(60+15+5) \mathrm{cm}$
$3 \mathrm{~s}-2 \mathrm{~s}=80 \mathrm{~cm}$
$\mathrm{s}=80 \mathrm{~cm}$
Area of $\triangle A B C$
$=\sqrt{s(s-a)(s-b)(s-c)}$
$\sqrt{80 \times 60 \times 15 \times 5}$
$\sqrt{20 \times 4 \times 20 \times 3 \times 5 \times 3 \times 5}$
$=20 \times 2 \times 3 \times 5 \mathrm{~cm}^{2}$
$=600 \mathrm{~cm}^{2}$
5. (B) $(2 \sqrt{2}+3 \sqrt{3})^{2}$
$(2 \sqrt{2})^{2}+2 \times 2 \sqrt{2}+3 \sqrt{3}+(3 \sqrt{3})^{2}$
$=8+12 \sqrt{6}+27$
$=35+12 \sqrt{6}$
6. (C) $\frac{1}{\sqrt{7}+\sqrt{6}-\sqrt{13}}$
$=\frac{1}{(\sqrt{7}+\sqrt{6})-(\sqrt{13})} \times \frac{(\sqrt{7}+\sqrt{6})+\sqrt{13}}{(\sqrt{7}+\sqrt{6})+(\sqrt{13})}$
$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{(\sqrt{7}+\sqrt{6})^{2}-(\sqrt{13})^{2}}$
$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{7+6+2 \sqrt{42}-13}$
$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{13+2 \sqrt{42}-13}$
$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2 \sqrt{42}}$
$=\frac{\sqrt{7}+\sqrt{6}+\sqrt{13}}{2 \sqrt{42}} \times \frac{\sqrt{42}}{\sqrt{42}}$
$=\frac{\sqrt{7} \times \sqrt{42}+\sqrt{6} \times \sqrt{42}+\sqrt{13} \times \sqrt{42}}{2(\sqrt{42})^{2}}$
$=\frac{7 \sqrt{6}+6 \sqrt{7}+\sqrt{546}}{2 \times 42}$
$=\frac{7 \sqrt{6}+6 \sqrt{7}+\sqrt{546}}{84}$
7. (C) Given $3 x+7^{\circ}+2 x-19^{\circ}+x=180^{\circ}$
$6 x=180^{\circ}+12^{\circ}$
$x=\frac{192^{\circ}}{6}=32^{\circ}$
$\therefore \quad \angle \mathrm{COD}=2 x-19^{\circ}=64^{\circ}-19^{\circ}=45^{\circ}$
8. (B) $x^{2}-y^{2}+z^{2}-p^{2}-2 y p-2 z x$

$$
\begin{aligned}
& =\left(x^{2}+z^{2}-2 z x\right)-\left(y^{2}+\mathrm{p}^{2}+2 y \mathrm{p}\right) \\
& =(x-z)^{2}-(y+\mathrm{p})^{2} \\
& =(x-z+y+\mathrm{p})(x-z-y-\mathrm{p})
\end{aligned}
$$

9. (C) $\operatorname{LHS}=6 x\left(x^{2}-4 y^{2}\right)-3 y\left(x^{2}-4 y^{2}\right)$
$=\left(x^{2}-4 y^{2}\right)(6 x-3 y)$
$=(2-2 y)(x+2 y)(3)(2 x-y)$
$=3(2 x-y)(x+2 y)(x-2 y)$
10. (D) Given $(x+1)$ is a factor of $\mathrm{p}(x)$
$=x^{2023}-3 x^{2022}+\mathrm{k}$
$p(-1)=0$
$p(-1)=(-1)^{2023}-3(-1)^{2022}+k=0$
$-1-3+k=0$
$k=4$
11. (A) Given $(2 x-3)$ is a factor of $p(x)$
$=2 x^{3}-x^{2}+m x+n$
$\therefore \mathrm{p}\left(\frac{3}{2}\right)=0$
$2\left(\frac{3}{2}\right)^{3}-\left(\frac{3}{2}\right)^{2}+m\left(\frac{3}{2}\right)+n=0$
$\Rightarrow 2 \times \frac{27}{8}-\frac{9}{4}+\frac{3 m}{2}+n=0$
$\frac{27-9+6 m+4 n}{4}=0$
$18+6 m+4 n=0 \times 4$
$6 m+4 n=-18$
$2(3 m+2 n)=-18$
$3 m+2 n=-9$
12. (C) Given $x=-3$ and $y=4$ is the solution of $5 x+3 y=k$
$\therefore \quad 5(-3)+3(4)=k$
$-15+12=k$
$k=-3$
13. (B) It is a right angled triangle of base 8 units and height 7 units

$\therefore \quad$ Area of $\triangle A O B=\frac{1}{2} \times$ bh $=\frac{1}{2} \times 8 \times 7$ sq. units $=28$ square units
14. (B) Consider the following figure


Using the exterior angle theorem, we get
$\angle y=\angle 1+\angle 2$
$\angle 1+\angle x=\angle 4$
Add equations (1) and (2)
$\angle y+\angle 1+\angle x=\angle 1+\angle 2+\angle 4$
$\Rightarrow \angle x+\angle y=\angle 2+\angle 4$
Now, since $\angle 2=\angle 4=\frac{z}{2}, x+y=z$
$\Rightarrow x=z-y$
15. (A) Consider the figure


Since thickness of the bowl is 0.25 cm , the outer radius R of the bowl is $5+0.25$ $=5.25 \mathrm{~cm}$

$$
\mathrm{V}_{\text {inner }}=\frac{2}{3} \pi \mathrm{r}^{3}=\frac{2}{3} \pi(5)^{3} \mathrm{~cm}^{3}
$$

$\therefore \quad$ Volume of steel used $=\mathrm{V}_{\text {outer }}-\mathrm{V}_{\text {inner }}$

$$
\begin{aligned}
& =\frac{2}{3} \pi(5.25)^{3}-\frac{2}{3} \pi(5)^{3} \\
& =\frac{2}{3} \times 3.14 \times\left(5.25^{3}-5^{3}\right) \\
& =41.25 \mathrm{~cm}^{3}
\end{aligned}
$$

16. (D) Since the height of the cylinder is 6 cm , the circumference of the base becomes 22 cm .


Let $r$ be the radius of its base. Then $2 \pi r=$ 22 cm
$\Rightarrow \mathrm{r}=\frac{7}{2} \mathrm{~cm}$
$\therefore \quad$ Volume of cylinder $=\pi r^{2} h=\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 6$ $=231 \mathrm{~cm}^{3}$
17. (A)

Given


In the figure, XZ coincides with $\mathrm{XY}+\mathrm{YZ}$. Also, Euclid's axiom (11) states that things which coincide with one another are equal to one another. So it is evident that $X Y+Y Z=X Z$
18. (C)

$\angle \mathrm{HKI}=25^{\circ}$
[Alternate angles since $A B \| C D$ ]
$\angle \mathrm{AIH}=\angle \mathrm{CHG}=60^{\circ}$
[Corresponding angles]
$\Rightarrow \angle \mathrm{JIA}=180^{\circ}-160^{\circ}=120^{\circ}$ [Linear pair]
$\angle \mathrm{LKI}=\angle \mathrm{JIA}=120^{\circ}$
[Corresponding angles since, GJ||KL]
$\angle \mathrm{HKL}=\angle \mathrm{HKI}+\angle \mathrm{LKI}=145^{\circ}$
19. (C) $a=\sqrt{5}, b=\sqrt[3]{7}$ and $c=\sqrt[4]{36}$
$a=5^{\frac{1}{2}}, b=7^{\frac{1}{3}}$ and $c=\left(6^{2}\right)^{\frac{1}{4}}=6^{\frac{1}{2}}$
$a=\left(5^{3}\right)^{\frac{1}{6}}, b=\left((7)^{2}\right)^{\frac{1}{6}}$ and $c=\left(6^{3}\right)^{\frac{1}{6}}$
$a=(125)^{\frac{1}{6}}, b=(49)^{\frac{1}{6}}$ and $c=(216)^{\frac{1}{6}}$
$\therefore \quad \mathrm{b}<\mathrm{a}<\mathrm{c}$
20. (C) $\sqrt{4 a^{2}+25 b^{2}+49 c^{2}-20 a b+70 b c-28 c a}$
$=\sqrt{(2 \mathrm{a})^{2}+(-5 b)^{2}+(-7 c)^{2}+2(2 a)(-5 b)+2(-5 b)(7 c)+2(-7 c)(2 a)}$
$=\sqrt{(2 a-5 b-7 c)^{2}}$
$=(2 a-5 b-7 c)$
21. (D) Given $\angle \mathrm{EAD}=20^{\circ} \Rightarrow \angle \mathrm{EOD}=2 \angle \mathrm{EAD}=40^{\circ}$


But $\triangle E O D \cong \triangle C O D[\because$ Given $D E=D C]$
$\Rightarrow \angle \mathrm{COD}=\triangle \mathrm{EOD}=40^{\circ}$

$$
\therefore \quad \angle \mathrm{BOD}=\angle \mathrm{BOD}+\angle \mathrm{COD}=100^{\circ}+40^{\circ}=140^{\circ}
$$

$\therefore \angle \mathrm{BAD}=\frac{\angle \mathrm{BOD}}{2}=\frac{140^{\circ}}{2}=70^{\circ}$
22. (D) Let $A B C D$ be the field in the form of a trapezium in which $A B \| C D$ such that

$A B=25 \mathrm{~m}, \mathrm{BC}=13 \mathrm{~cm}, C D=10 \mathrm{~cm}$ and $D A=14 \mathrm{~m}$

Draw CE || DA and CF $\perp$ EB

Clearly, $A B C E$ is a parallelogram.
$\therefore \quad C E=D A=14 \mathrm{~m}$ and $A E=C D=10 \mathrm{~m}$
$\therefore \quad E B=A B-A E=(25-10) m=15 m$
In $\triangle E B C$, we have
$E B=15 \mathrm{~m}, \mathrm{BC}=13 \mathrm{~m}$ and $C E=14 \mathrm{~m}$
$\therefore \quad a=15 \mathrm{~m}, \mathrm{~b}=13 \mathrm{~m}$ and $\mathrm{c}=14 \mathrm{~m}$
$\therefore \quad s=\frac{1}{2}(15+13+14) m=21 m$
$\therefore \quad(s-a)=(21-15) m=6 m$
$(s-b)=(21-13) m=8 m$ and
$(s-c)=(21-14) m=7 m$
$\therefore \quad$ area $(\triangle E B C)=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{21 \times 6 \times 8 \times 7} \mathrm{~m}^{2}$
$=\sqrt{7 \times 3 \times 3 \times 2 \times 2 \times 4 \times 7} \mathrm{~m}^{2}$
$=(7 \times 3 \times 2 \times 2) \mathrm{m}^{2}=84 \mathrm{~m}^{2}$
Also area ( $\triangle \mathrm{EBC}$ )
$=\left(\frac{1}{2} \times E B \times C F\right)=\left(\frac{1}{2} \times 15 \mathrm{~m} \times \mathrm{CF}\right)$
$\therefore \quad \frac{1}{2} \times 15 \mathrm{~m} \times \mathrm{CF}=84 \mathrm{~m}^{2}$
$\Rightarrow \mathrm{CF}=\frac{84 \times 2}{15} \mathrm{~m}=\frac{56}{5} \mathrm{~m}=11.2 \mathrm{~m}$
$\therefore \quad C F=11.2 \mathrm{~m}$
Area (trap. $A B C D)=\frac{1}{2} \times(A B+C D) \times C F$
$=\left\{\frac{1}{2} \times(25+10) \times 11.2\right\} \mathrm{m}^{2}$
$=(35 \times 5.6) \mathrm{m}^{2}=196 \mathrm{~m}^{2}$
Hence, area of trapezium $A B C D$ is $196 \mathrm{~m}^{2}$
23. (D) Radius of each hemispherical end $=7 \mathrm{~cm}$


Hence of each hemispherical part = its radius $=7 \mathrm{~cm}$

Height of the cylindrical part
$=(104-2 \times 7) \mathrm{cm}=90 \mathrm{~cm}$
Area of surface to be polished
= 2(curved surface area of the hemisphere)

+ (curved surface area of the cylinder)
$=\left[2\left(2 \pi r^{2}\right)+2 \pi r h\right]$ sq. units
$=\left[\left(4 \times \frac{22}{7} \times 7 \times 7\right)+\left(2 \times \frac{22}{7} \times 7 \times 90\right)\right] \mathrm{cm}^{2}$
$=(616+3960) \mathrm{cm}^{2}=4576 \mathrm{~cm}^{2}$
$=\left(\frac{4576}{10 \times 10}\right) \mathrm{dm}^{2}=45.76 \mathrm{dm}^{2}$
$[\because 10 \mathrm{~cm}=1 \mathrm{dm}]$
$\therefore \quad$ cost of polishing the surface of the solid $=₹(45.76 \times 10)=₹ 457.60$

24. (A) $30^{\circ}+\alpha_{1}=\alpha, 45^{\circ}+\alpha_{2}=\beta$


Adding $\left(30^{\circ}+45^{\circ}\right)+\left(\alpha_{1}+\alpha_{2}\right)=\alpha+\beta$
$\Rightarrow 75^{\circ}+55^{\circ}$
$=x^{\circ} \Rightarrow x=130^{\circ}$
25. (B) $\angle \mathrm{ACD}+80^{\circ}=180^{\circ}$
$\Rightarrow \angle \mathrm{ACD}=100^{\circ}$
$\angle \mathrm{ECF}=\angle \mathrm{ACD}=100^{\circ}$
$100^{\circ}+25^{\circ}+\angle \mathrm{CEF}=180^{\circ}$
$\Rightarrow \angle \mathrm{CEF}=55^{\circ}$
26. (A) We know that the perpendicular from the centre of a circle to a chord bisects the chord.
$\therefore A P=\frac{1}{2} A B=\left(\frac{1}{2} \times 8\right) \mathrm{cm}=4 \mathrm{~cm}$
$\mathrm{CQ}=\frac{1}{2} \mathrm{CD}=\left(\frac{1}{2} \times 6\right) \mathrm{cm}=3 \mathrm{~cm}$
Join OA and OC
Then, $O A=O C=5 \mathrm{~cm}$
From the right-angled $\triangle$ OPA, we have $\mathrm{OP}^{2}=\mathrm{OA}^{2}-\mathrm{AP}^{2}=\left[(5)^{2}-(4)^{2}\right] \mathrm{cm}^{2}=9 \mathrm{~cm}^{2}$
$\Rightarrow O P=3 \mathrm{~cm}$
From the right-angled $\triangle \mathrm{OQC}$, we have
$\mathrm{OQ}^{2}=\mathrm{OC}^{2}-\mathrm{CQ}^{2}=\left[(5)^{2}-(3)^{2}\right] \mathrm{cm}^{2}=16 \mathrm{~cm}^{2}$
$\Rightarrow O Q=4 \mathrm{~cm}$
Since $O P \perp A B, O Q \perp C D$ and $A B \| C D$, the points $P, O, Q$ are collinear.
$\therefore \quad P Q=O P+O Q=(3+4) \mathrm{cm}=7 \mathrm{~cm}$
27. (A) Since ACE is a striaght line, we have
$\angle \mathrm{ACB}+\angle \mathrm{BCE}=180^{\circ}$
$\Rightarrow 90^{\circ}+\angle \mathrm{BCE}=180^{\circ}$
$[\because \angle A C B$ is in a semicircle]
$\Rightarrow \angle \mathrm{BCE}=90^{\circ}$
Also, $\angle \mathrm{DBC}=\frac{1}{2} \angle \mathrm{COD}=\left(\frac{1}{2} \times 40^{\circ}\right)=20^{\circ}$
[angle at centre $=2 \times$ angle at a point on a circle]
$\Rightarrow \angle \mathrm{EBC}=\angle \mathrm{DBC}=20^{\circ}$
Now, in $\angle E B C$, we have
$\angle \mathrm{EBC}+\angle \mathrm{BCE}+\angle \mathrm{CEB}=180^{\circ}$
$\Rightarrow 20^{\circ}+90^{\circ}+\angle \mathrm{CED}=180^{\circ}$
$[\because \angle C E B=\angle C E D]$
$\Rightarrow \angle C E D=180^{\circ}-110^{\circ}=70^{\circ}$
Hence, $\angle \mathrm{CED}=70^{\circ}$
28. (B) Given $\mathrm{a}=169 \mathrm{~cm}, \mathrm{~b}=174 \mathrm{~cm} \& \mathrm{c}=245 \mathrm{~cm}$
$\therefore \mathrm{s}=\frac{\mathrm{a}+\mathrm{b}+\mathrm{c}}{2}=\frac{588}{2} \mathrm{~cm}=294 \mathrm{~cm}$
Area of the triangle
$=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{294 \times 125 \times 120 \times 49} \mathrm{~cm}^{2}$
$=\sqrt{7 \times 7 \times 6 \times 5 \times 5 \times 5 \times 5 \times 6 \times 4 \times 49} \mathrm{~cm}^{2}$
$=7 \times 6 \times 5 \times 5 \times 2 \times 7 \mathrm{~cm}^{2}$
$=14700 \mathrm{~cm}^{2}$
29. (C) Given $\angle B A D+\angle B C D=180^{\circ}$
$[\because$ Given $A B C D$ is a cyclic quadrilateral]
$x-y+x+y=180^{\circ}$
$2 x=180^{\circ}$
$x=90^{\circ}$
30. (A) Given $x=\frac{5-\sqrt{21}}{2}$
$\therefore \frac{1}{x}=\frac{2}{5-\sqrt{21}}=\frac{2}{(5-\sqrt{21})} \times \frac{(5+\sqrt{21})}{(5+\sqrt{21})}$
$=\frac{2(5+\sqrt{21})}{(5)^{2}-(\sqrt{21})^{2}}$
$=\frac{2(5+\sqrt{21})}{(25-21)}=\frac{2(5+\sqrt{21})}{4}=\frac{5+\sqrt{21}}{2}$
$\therefore x+\frac{1}{x}=\left(\frac{5-\sqrt{21}}{2}\right)+\left(\frac{5+\sqrt{21}}{2}\right)$
$=\frac{5-\sqrt{21}+5+\sqrt{21}}{2}=\frac{10}{2}=5$
$\therefore\left(x+\frac{1}{x}\right)^{2}=5^{2}=25$
$\Rightarrow x^{2}+\frac{1}{x^{2}}+2 \times x \times \frac{1}{x}=25$

$$
\Rightarrow\left(x^{2}+\frac{1}{x^{2}}\right)+2=25 \Rightarrow\left(x^{2}+\frac{1}{x^{2}}\right)=25-2=23
$$

$$
\text { And, } x+\frac{1}{x}=5 \Rightarrow\left(x+\frac{1}{x}\right)^{3}=(5)^{3}=125
$$

$$
x^{3}+\frac{1}{x^{3}}+3 \times x \times \frac{1}{x}\left(x+\frac{1}{x}\right)=125
$$

$$
\Rightarrow\left(x^{3}+\frac{1}{x^{3}}\right)+3 \times 5=125
$$

$$
\Rightarrow\left(x^{3}+\frac{1}{x^{3}}\right)=125-15=110
$$

$$
\therefore\left(x^{3}+\frac{1}{x^{3}}\right)-5\left(x^{2}+\frac{1}{x^{2}}\right)+\left(x+\frac{1}{x}\right)
$$

$$
=110-5 \times 23+5=110-115+5
$$

$$
=115-115=0
$$

## MATHEMATICS - 2

31. (A, D)
$\pi$ is irrational
$\Rightarrow$ Surface area and volume of the sphere also irrational numbers
32. (A, B, C, D)

Option (A) : If $a+b+c=0$, then $a^{3}+b^{3}+c^{3}=3 a b c$
Option (B) : Given $\mathrm{a}+\mathrm{b}+\mathrm{c}=0$
squaring on both sides $(a+b+c)^{2}=0$
$a^{2}+b^{2}+c^{2}+2(a b+b c+c a)=0$
$\therefore a^{2}+b^{2}+c^{2}=-2(a b+b c+c a)$
Option (C) : Given $a+b+c=0$
$\therefore \mathrm{c}=-(\mathrm{a}+\mathrm{b})$
If $a+b+c=0$, then
$a^{3}+b^{3}+c^{3}=3 a b c=-3 a b(a+b)$
Option (D) : Given $a+b+c=0$
$\therefore(a+b)^{2}=(-c)$
squaring on both sides
$(a+b)^{2}=(-c)^{2}$
$a^{2}+b^{2}+2 a b=c^{2}$
$\therefore a^{2}+b^{2}-c^{2}=-2 a b$
33. $(A, B)$

Given $\mathrm{f}(x)=2 x^{100}-19 x^{99}+8 x^{98}+19 x^{95}-10 x^{92}$ If $\mathrm{f}(x)$ is divided by $(x-1)$ then the remainder is $f(1)$
$\therefore f(1)=2 \times 1^{100}-19(1)^{99}+8(1)^{98}+19(1)^{95}-10(1)^{92}$
$=2-19+8+19-10$
$=0$
$f(1)=0 \Rightarrow(x-1)$ is a factor of $f(x)$
$f(-1)=2(-1)^{100}-19(-1)^{99}+8(-1)^{98}+19(-1)^{95}-10(-1)^{92}$
$2 \times 1-19 \times-1+8(1)+19(-1)-10(1)$
$=2+19+8-19-10$
$f(1)=0 \Rightarrow(x+1)$ is a factor of $f(x)$
$f(-2)=2(-2)^{100}-19(-2)^{99}+8(-2)^{98}+19(-2)^{95}-10(-2)^{92}$
$=2 \times 2^{100}+19 \times 2^{99}+8 \times 2^{98}-19 \times 2^{95}-10 \times 2^{92}$
$\therefore \mathrm{f}(-2) \neq 0 \Rightarrow(x+2)$ is not a factor of $\mathrm{f}(x)$
Similarly $(x-2)$ is not a factor of $f(x)$
34. (A, B, D)

Every square is also a parallelogram
Every rectangle is also a parallelogram
Every rhombus is also a parallelogram
35. (A, C, D)

Option $A \times \sqrt[5]{a^{8} b^{7} c^{4}}=\sqrt[5]{a^{7} b^{3} c} \times \sqrt[5]{a^{8} b^{7} c^{4}}$
$\sqrt[5]{a^{7} \times a^{8} \times b^{3} \times b^{7} \times c \times c^{4}}$
$\sqrt[5]{a^{15} b^{10} c^{5}}$
$=a^{3} b^{2} c$ which is a rational number
$\therefore \sqrt[5]{a^{7} b^{3} c}$ is the RF of $\sqrt[5]{a^{8} b^{7} c^{4}}$
Option $B \times \sqrt[5]{a^{8} b^{7} c^{4}}=\sqrt{a^{2} b^{3} c} \times \sqrt[5]{a^{8} b^{7} c^{4}}$
Orders of both surds are not same.
If we made orders are same also we cannot get rational number.
$\therefore \sqrt{a^{2} b^{3} c}$ is not a RF of $\sqrt[5]{a^{8} b^{7} c^{4}}$
Similarly we can prove $\sqrt[5]{a^{2} b^{8} c^{6}}$ and $\sqrt[5]{a^{7} b^{8} c^{11}}$ are also RF of $\sqrt[5]{a^{8} b^{7} c^{4}}$

## REASONING

36. (D) $1=d, 2=c$

37. (C) In all the others the outer figure is repeated in the middle.

38. (A) Silk (Mohair is type of wool, where as shanting is type of silk.
39. (C) We have
$15 \times 2=30,2 \times 7=14,7 \times 9=63$
So, missing number $=9 \times 15=135$
40. (B) Odds are successor and even are predecessor.
41. (A)


42. (B)

43. (C)

44. (B)

45. (D)

46. (C)

47. (A) Both the koala and the weight go up with equal acceleration.
48. (A)


If the pattern follows $C, A$ and $B$ switches
one by one the result figure

lights 1 and 2
are in reverse order. So, switch (A) is fault.
49. (B) Option (B) is correct because the argument 1 states that banning pesticides is the only way to save underground water but we know that it is not the only way. Other measures can also be taken to reduce the pollution.
50. (B) The steepest lines on the graph represent the thinnest part of the container, as this is a part of the container that would become fuller quicker. This graph represents a container that starts thin, gradually becomes wider and then becomes thin again at the top.

